The following is the parameter file web5.par. The data is also included in this file. In this example, the dependent variable *y* is a vector with two terms: *y1* and *y2*. The mathematical model is based upon the following two coupled differential equations:

$$\frac{dy_1}{dt} = -a1(y_1 - y_2)$$
$$\frac{dy_2}{dt} = a3(y_1 - y_2)^{a2}$$

This is an initial value problem in which *y1*=100 and *y2*=0 at time *t*=0. The values of *a*1, *a*2 and *a*3 can be determined by the method of least squares using the values of *t*, *y1* and *y2* included in the file. The two differential equations are first transformed into the following integral equations:

$$y_1 = 100 - a1\int_0^t (y_1 - y_2)dt$$
$$y_2 = a3\int_0^t (y_1 - y_2)^{a2}dt$$

Using the INT operator these equations can now be modeled in REGRESS. Note that these equations are recursive in the sense that *y1* is dependent upon *y2* and *y2* is dependent upon *y1*.

```
! A nonlinear recursive model using the INT operator
   y1 = 'a1 * int(y1-y2, 0, t) + 100'
   y2 = 'a3 * int((y1-y2)^a2, 0, t)'
   a0[1]=-1 a0[2]=0.5 a0[3]=1
   ncol=3  tcol=1  ycol1=2  ycol2=3
   caf=0.3
;
!   t       y1      y2
  0.0   100       0
  0.2    92.860   1.803
  0.4    85.498   3.425
  0.6    79.265   4.984
  0.8    73.626   6.194
  1.0    68.225   7.469
  1.2    63.905   8.524
  1.4    59.727   9.468
  1.6    55.946  10.413
  1.8    52.525  11.269
  2.0    49.430  11.742
;
```

The following is the output file web5.out. The information included in this file is also shown on the screen. To run this example the following command is issued from a DOS window:

```
regress web5
```

The convergence acceleration factor (caf) was reduced to 0.3 from the default value of 1. This reduced the number of iterations required from 75 to 8 (as seen in the output file below). Normally one only varies caf when there are convergence problems. However, for this example I reduced it for aesthetic reasons (i.e., to reduce the size of the output file). I should add that this parameter doesn't always reduce the number of iterations and often increases the number of iterations. However, for this example the effect was dramatic.

```
PARAMETERS USED IN REGRESS ANALYSIS: Wed Nov 01 16:02:33 2006

  INPUT PARMS FILE: web5.par
  INPUT DATA  FILE: web5.par
  REGRESS  VERSION: 4.16, Oct 31, 2006

  STARTREC - First record used            :     1
  N - Number of recs used to build model  :    11
  NO_DATA - Code for dependent variable    -999.0
  NCOL - Number of data columns           :     3
  NY   - Number of dependent variables    :     2
  YCOL1 - Column for dep var Y1              :  2
  YCOL2 - Column for dep var Y2              :  3
  SYTYPE1 - Sigma type for Y1             :     1
     TYPE 1:  SIGMA Y1 = 1
  SYTYPE2 - Sigma type for Y2             :     1
     TYPE 1:  SIGMA Y2 = 1
  M - Number of independent variables     :     1
  Column for X1                           :     1
  STTYPE1 - Sigma type for X1             :     0
     TYPE 0:  SIGMA X1 = 0
  NUMRECIT - Max iterations in recursion  :    10
  RAF - Recursion acceleration factor     :  1.00
  RECEPS - Recursion converge criterion   0.00100
  MAXDEPTH - Max depth in INT scheme      :    10
  INTEPS - Integration converge criterion 0.00100

Analysis for Set 1
  Function Y1:  A1 * INT(Y1-Y2, 0, T) + 100
  Function Y2:  A3 * INT((Y1-Y2)^A2, 0, T)

  EPS - Convergence criterion          : 0.00100
  CAF - Convergence acceleration factor :   0.300

 ITERATION          A1          A2          A3  S/(N.D.F.)
         0    -1.00000     0.50000     1.00000   360.83183
         1    -0.75608     0.67542     0.28788   197.78887
         2    -0.46967     0.89256     0.11603    15.16522
         3    -0.39607     1.07283     0.06165     0.34935
         4    -0.39737     1.06185     0.07157     0.02046
         5    -0.39752     1.06196     0.07172     0.01972
         6    -0.39758     1.06200     0.07178     0.01956
         7    -0.39762     1.06204     0.07182     0.01947
         8    -0.39765     1.06207     0.07184     0.01943
```

REC	Y-INDEX	X1	Y	SIGY	YCALC
1	1	0.00000	100.00000	1.00000	100.00000
2	1	0.20000	92.86000	1.00000	92.43096
3	1	0.40000	85.49800	1.00000	85.56903
4	1	0.60000	79.26500	1.00000	79.32393
5	1	0.80000	73.62600	1.00000	73.67630
6	1	1.00000	68.22500	1.00000	68.55788
7	1	1.20000	63.90500	1.00000	63.91854
8	1	1.40000	59.72700	1.00000	59.71297
9	1	1.60000	55.94600	1.00000	55.90017
10	1	1.80000	52.52500	1.00000	52.44308
11	1	2.00000	49.43000	1.00000	49.30815

REC	Y-INDEX	X1	Y	SIGY	YCALC
1	2	0.00000	0.00000	1.00000	0.00000
2	2	0.20000	1.80300	1.00000	1.81448
3	2	0.40000	3.42500	1.00000	3.44874
4	2	0.60000	4.98400	1.00000	4.92487
5	2	0.80000	6.19400	1.00000	6.25165
6	2	1.00000	7.46900	1.00000	7.44681
7	2	1.20000	8.52400	1.00000	8.52352
8	2	1.40000	9.46800	1.00000	9.49365
9	2	1.60000	10.41300	1.00000	10.36785
10	2	1.80000	11.26900	1.00000	11.15569
11	2	2.00000	11.74200	1.00000	11.86580

K	A0(K)	AMIN(K)	AMAX(K)	A(K)	SIGA(K)
1	-1.00000	Not Spec	Not Spec	-0.39767	0.00070863
2	0.50000	Not Spec	Not Spec	1.06209	0.06505
3	1.00000	Not Spec	Not Spec	0.07186	0.02003

Variance Reduction: 99.98 (Average)
VR: Y1 99.99
VR: Y2 99.97

S/(N - P) : 0.01941

RMS (Y - Ycalc) : 0.12948 (all data)
RMS(Y1-Ycalc): 0.17320
RMS(Y2-Ycalc): 0.05945

Runs Test for Y1: Number runs = 3 Must be > 2 to pass test.
This limit is based upon 2.5% confidence level.
Average number of runs if residuals random: 6.5.

Runs Test for Y2: Number runs = 7 Must be > 2 to pass test.
This limit is based upon 2.5% confidence level.
Average number of runs if residuals random: 6.5.